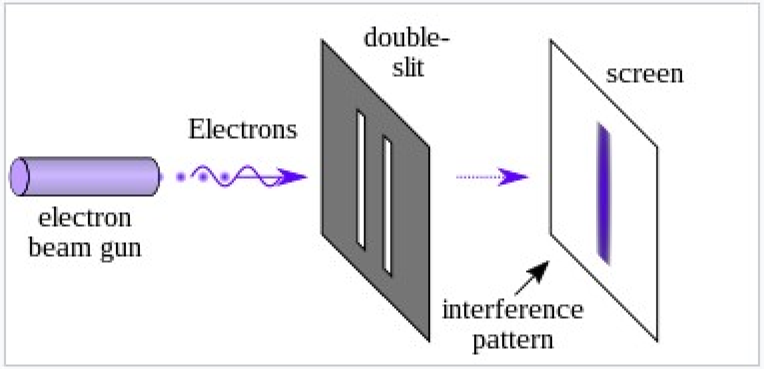
Brief overview of post-quantum cryptography
CSE 548 Spring 2024

jedimaestro@asu.edu

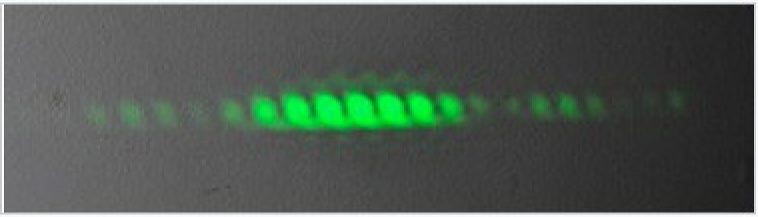
Some videos...

- https://www.youtube.com/watch?v=_C5dkUiiQnw
- https://www.youtube.com/watch?v=QDdOoYdb748
- https://www.youtube.com/watch?v=K026C5YaB3A

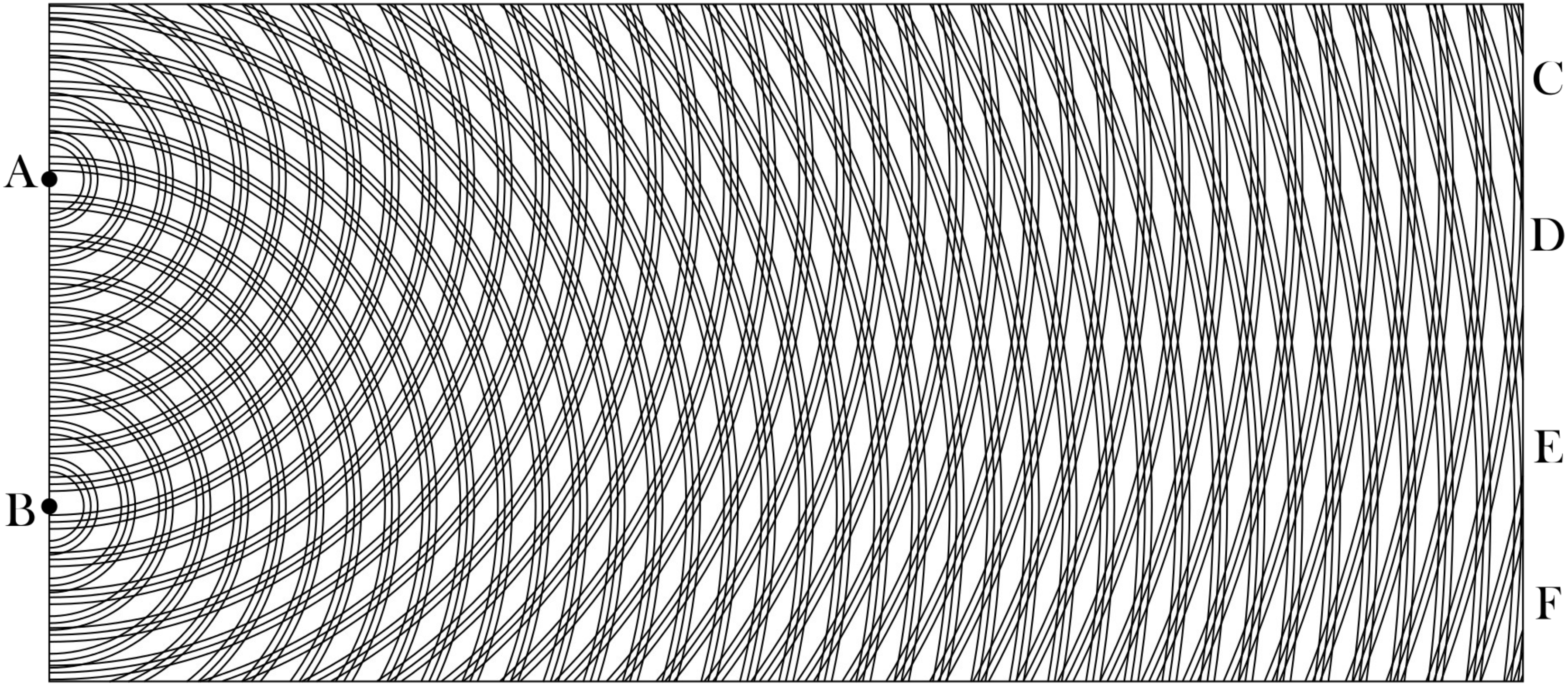
https://en.wikipedia.org/wiki/Double-slit_experiment

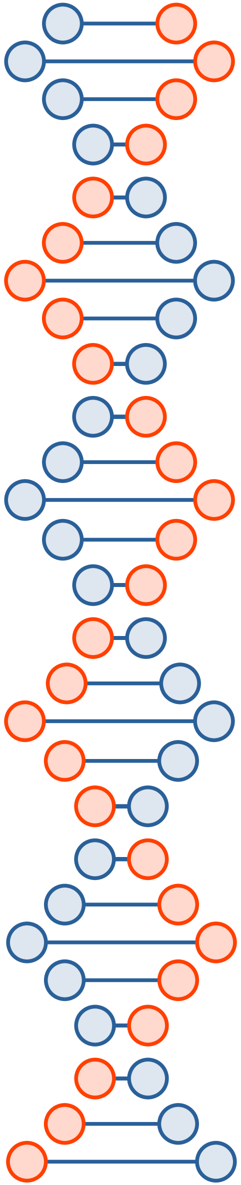


Photons or matter (like electrons) produce an interference pattern when two slits are used

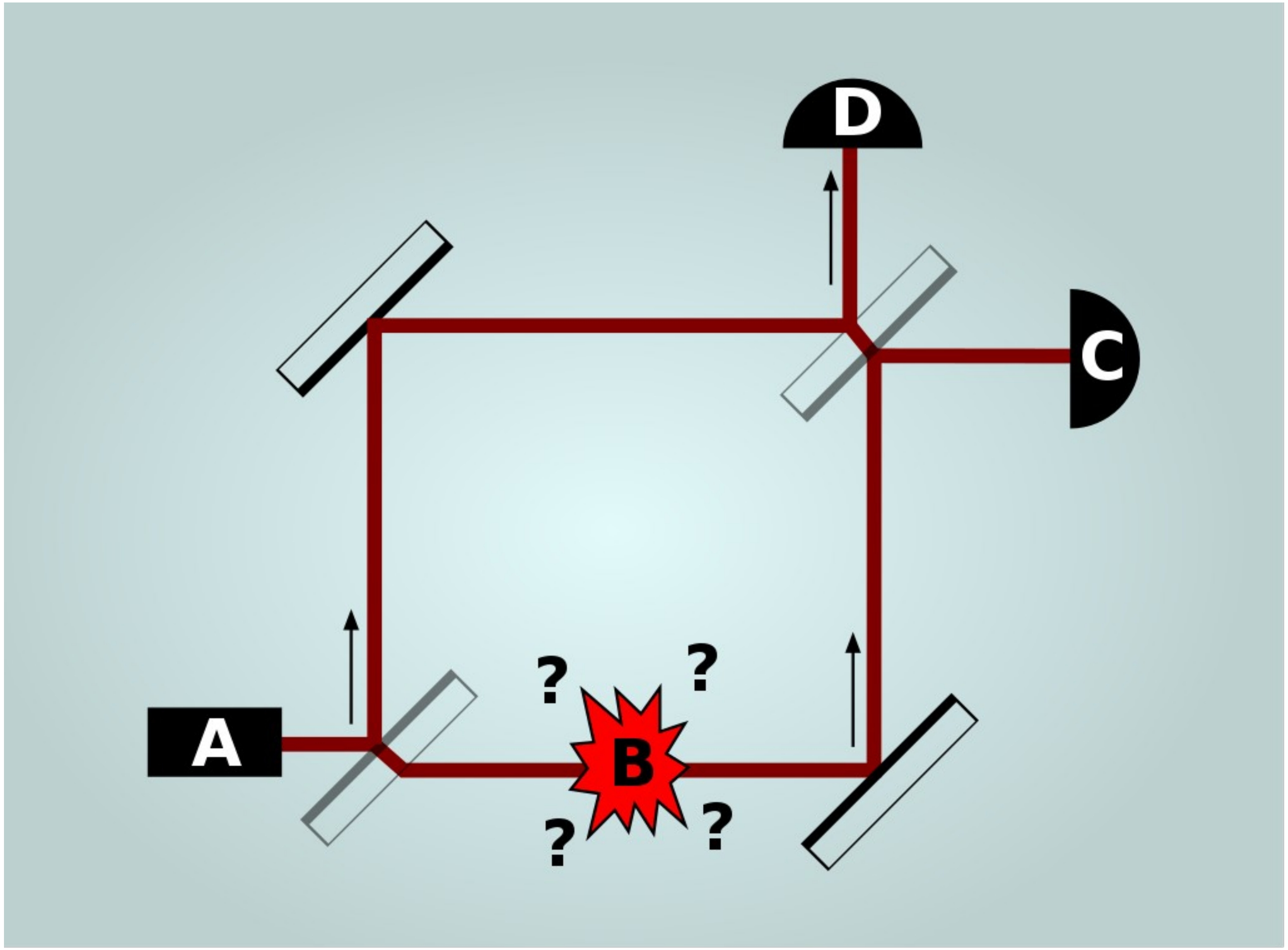


Light from a green laser passing through two slits 0.4mm wide and 0.1mm apart https://en.wikipedia.org/wiki/Double-slit_experiment

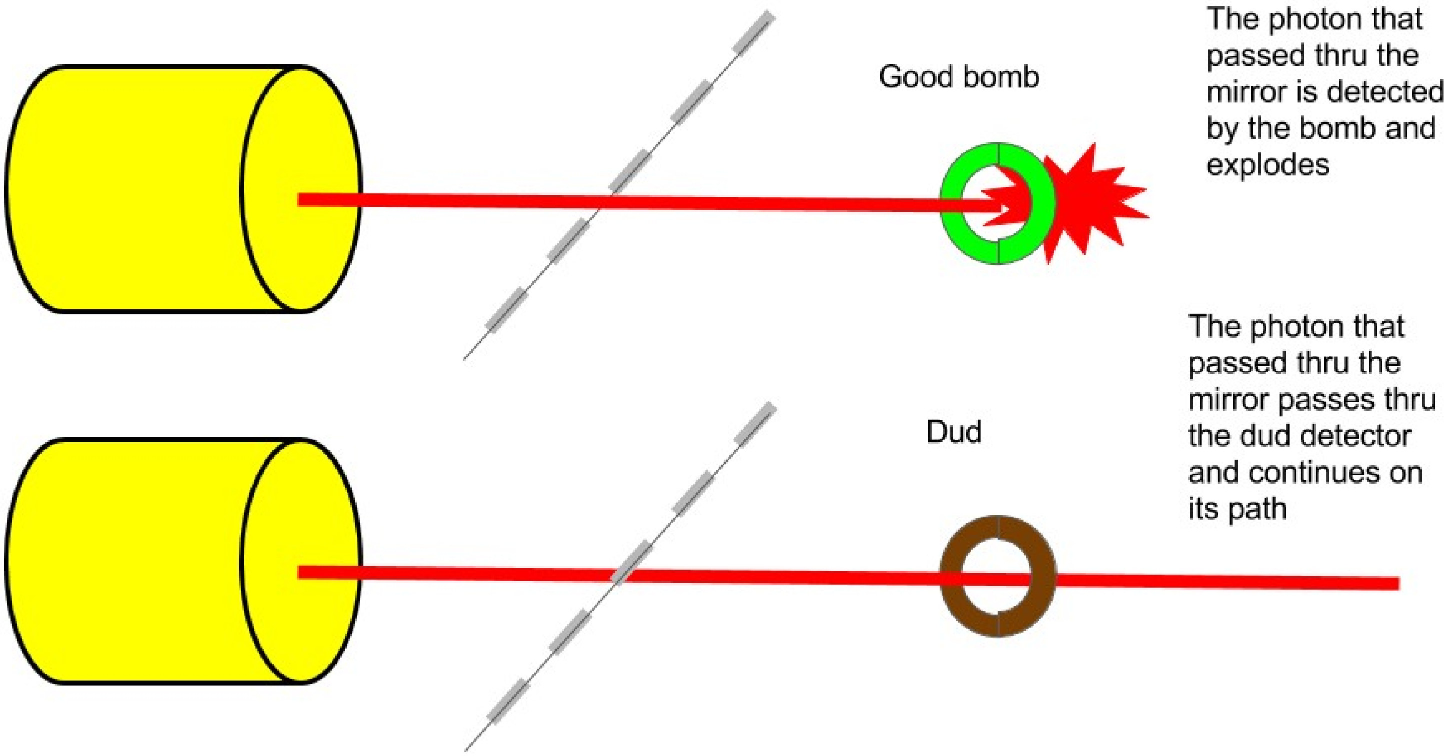




https://en.wikipedia.org/wiki/Elitzur%E2%80%93Vaidman_bomb_tester

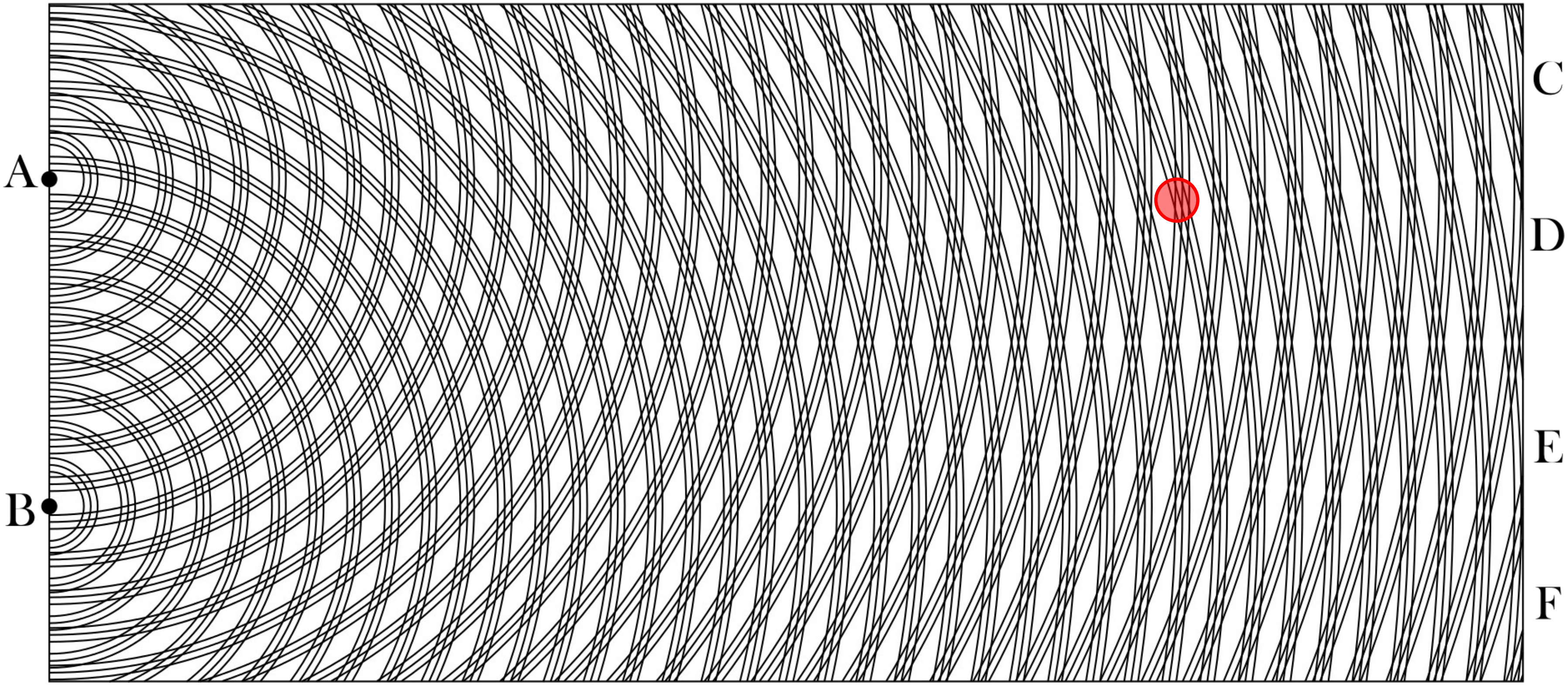


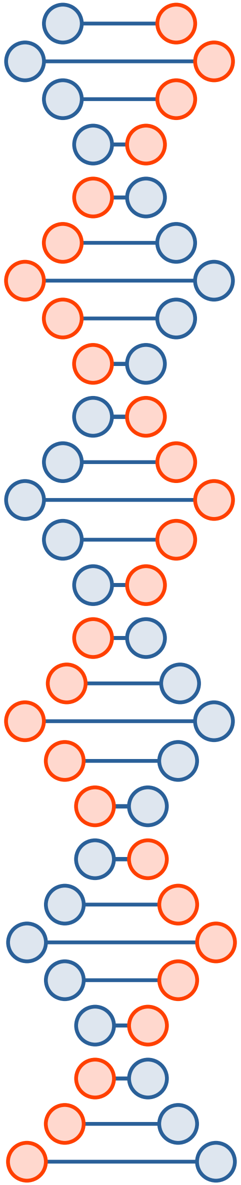
Bomb is either live or a dud



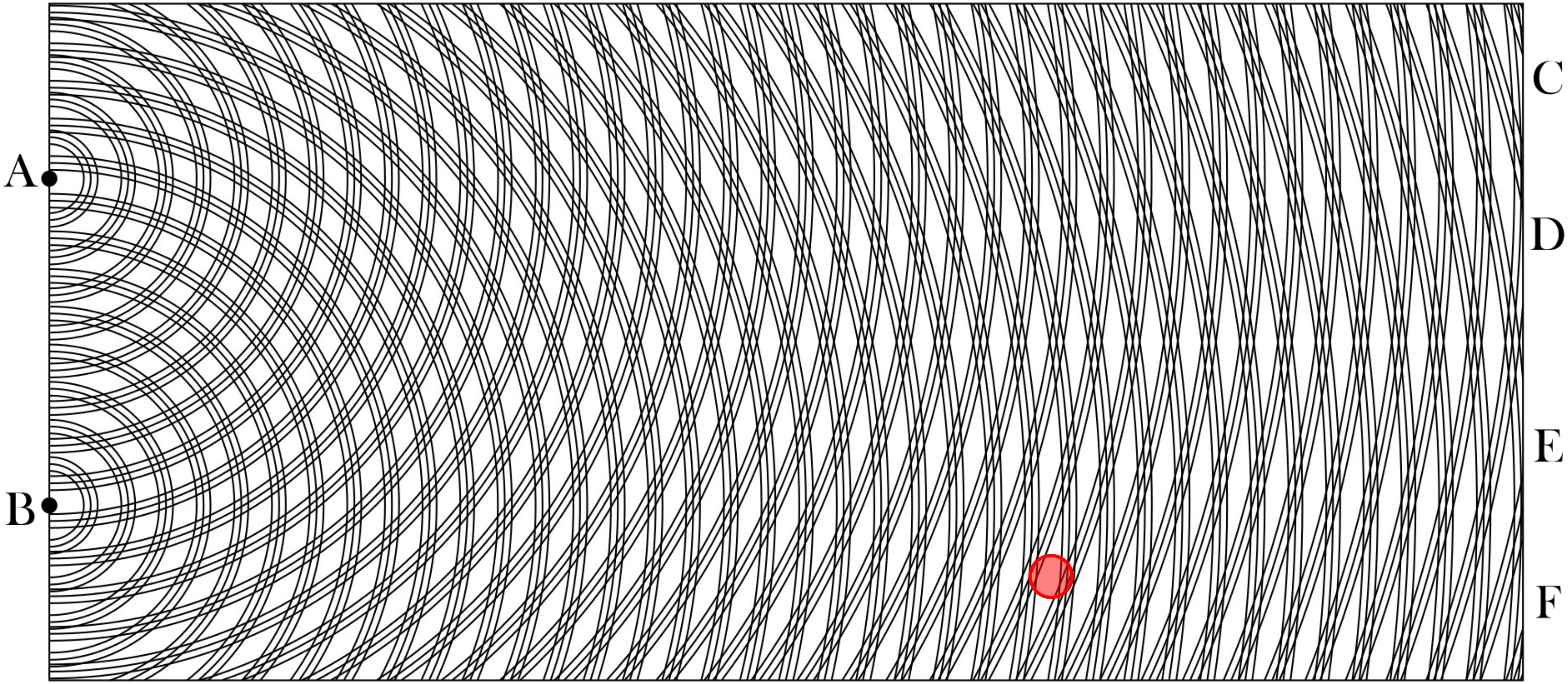
"Due to the way in which the interferometer is constructed, a photon going through the second mirror from the lower path towards detector D will have a phase shift of half a wavelength compared to a photon being reflected from the upper path towards that same detector..."

Put C, *e.g.*, here...

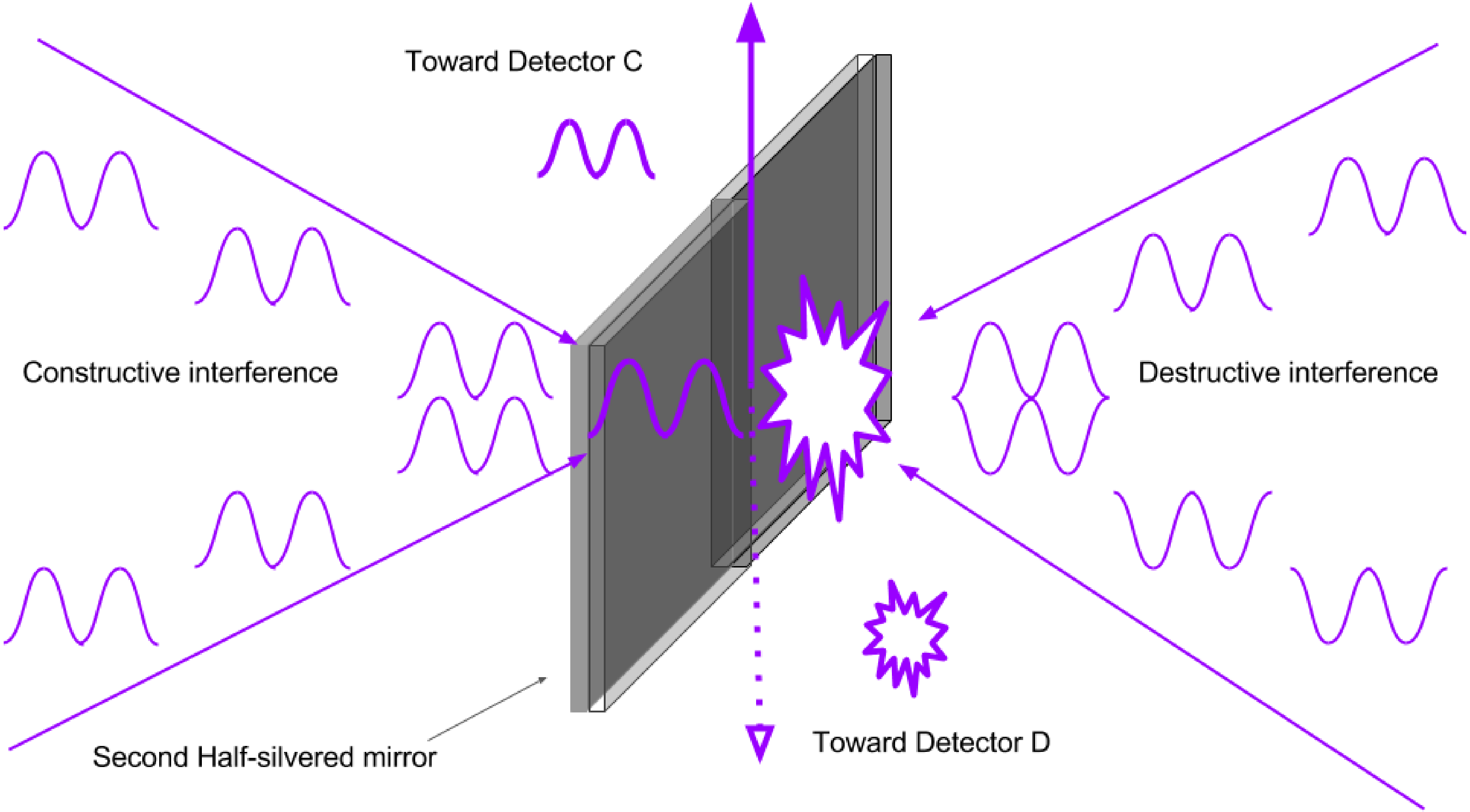




Put D, *e.g.*, here...



If both waves make it to the end (dud!)...



We will never detect a photon at D if the bomb is a dud.

(*I.e.*, if we detect a photon at D then the bomb is not a dud.)

Case #1: Bomb is a dud

- Experiment will keep showing a photon detected at C
- Keep repeating until we're as sure as we want to be that the bomb is in fact a dud

Case #2: Bomb is live

- 50% chance photon takes the lower path
    - Boom!
- 50% chance the photon takes the upper path
    - 50% chance (25% conditional) that the single photon (no longer a wave) goes to detector C
        - Have to repeat
    - 50% chance (25% conditional) that the single photon (no longer a wave) goes to detector D

Bomb is live (keep repeating)

- 2/3rds chance we blow ourselves up
- 1/3rd chance we eventually detect a photon at D
  - No boom, but we’re certain the bomb is live

WTF?

- With a decent probability (1/3), we learn information about something that *could have happened but didn't*.
- Interaction free experiment
  - Possible in classical physics, *e.g.*, I give you two envelopes and tell you a letter is in one and the other is empty, if you open one you know something about the other.
  - At quantum scales the letter is in a superposition of both envelopes until you observe it
    - These probabilities can be entangled

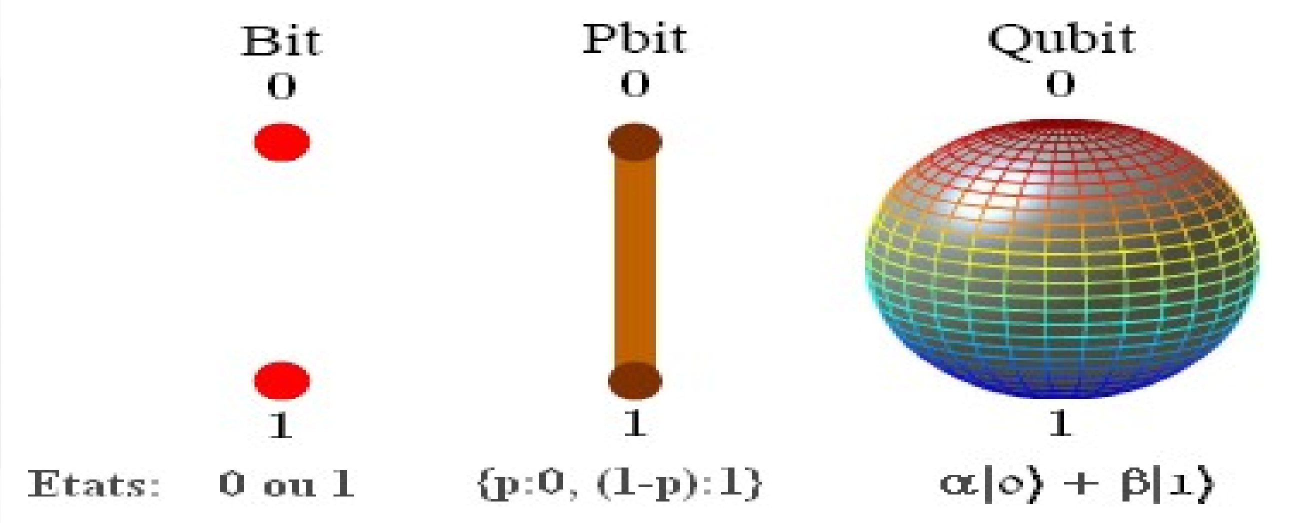
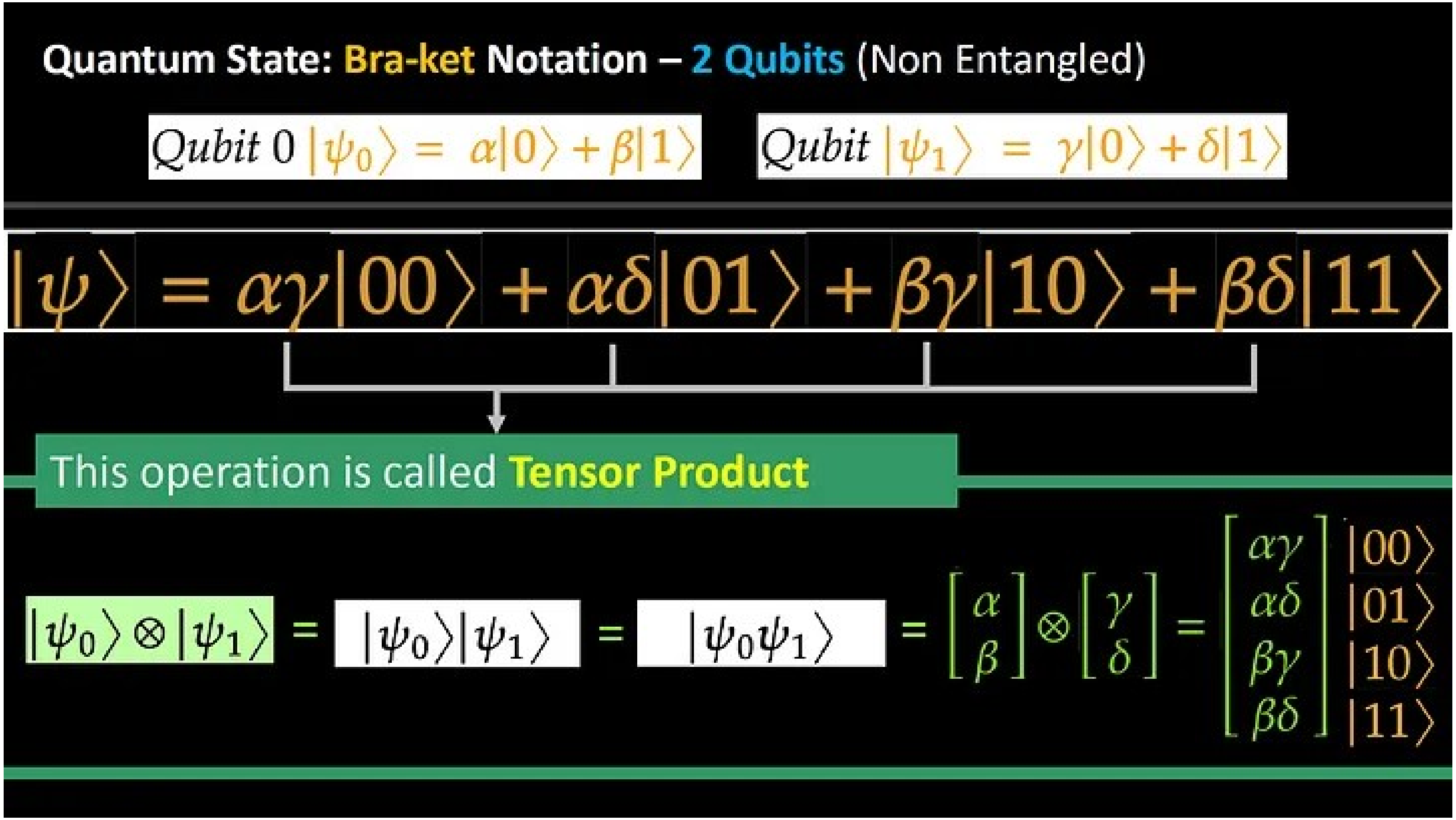


Image taken from http://filipchsqroom.blogspot.com/

Is superposition enough?

- As far as I know (but actual physicists are not in complete agreement on this) qubits have to be mutually entangled in very specific ways to implement useful quantum computations
  - Quantum decoherence is a major challenge



https://andisama.medium.com/qubit-an-intuition-2-inner-product-outer-product-and-tensor-product-in-bra-ket-notation-9d598cbd6bc

History of Quantum Entanglement

$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$$|\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

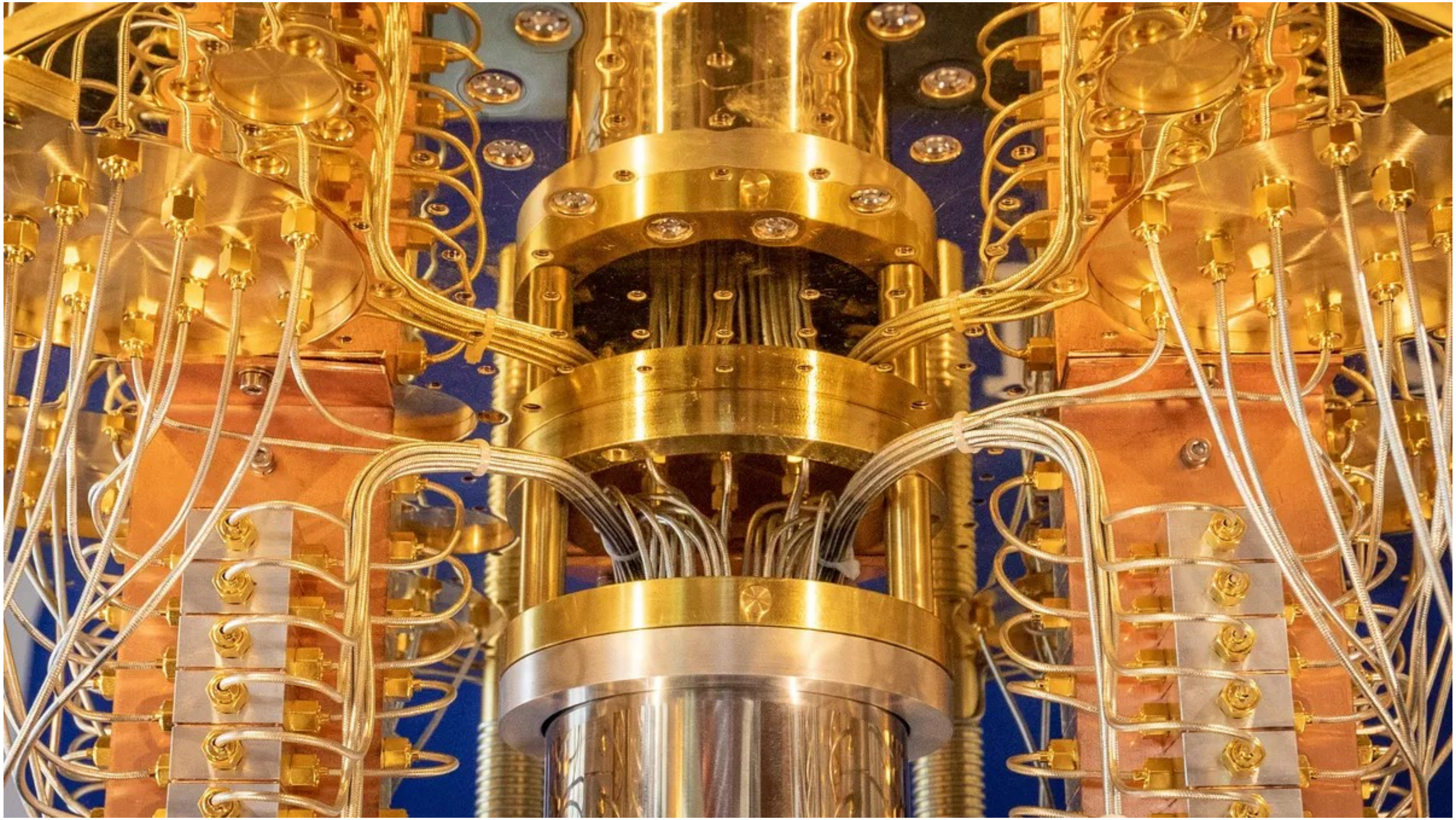
- Normalized: $\langle\beta_{00}|\beta_{00}\rangle = \frac{\langle 00| + \langle 11|}{\sqrt{2}} \cdot \frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(1+1) = 1$
- Orthogonal : $\langle\beta_{01}|\beta_{00}\rangle = \frac{\langle 01| + \langle 10|}{\sqrt{2}} \cdot \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

$$= \langle 0|0\rangle\langle 1|0\rangle + \langle 0|1\rangle\langle 1|1\rangle + \langle 1|0\rangle\langle 0|0\rangle + \langle 1|1\rangle\langle 0|1\rangle = 0$$

- Expansion:

$$|\alpha\beta\rangle = |\beta_{00}\rangle\langle\beta_{00}|\alpha\beta\rangle + |\beta_{01}\rangle\langle\beta_{01}|\alpha\beta\rangle + |\beta_{10}\rangle\langle\beta_{10}|\alpha\beta\rangle + |\beta_{11}\rangle\langle\beta_{11}|\alpha\beta\rangle$$

https://galileo-unbound.blog/2022/11/26/a-short-history-of-quantum-entanglement/

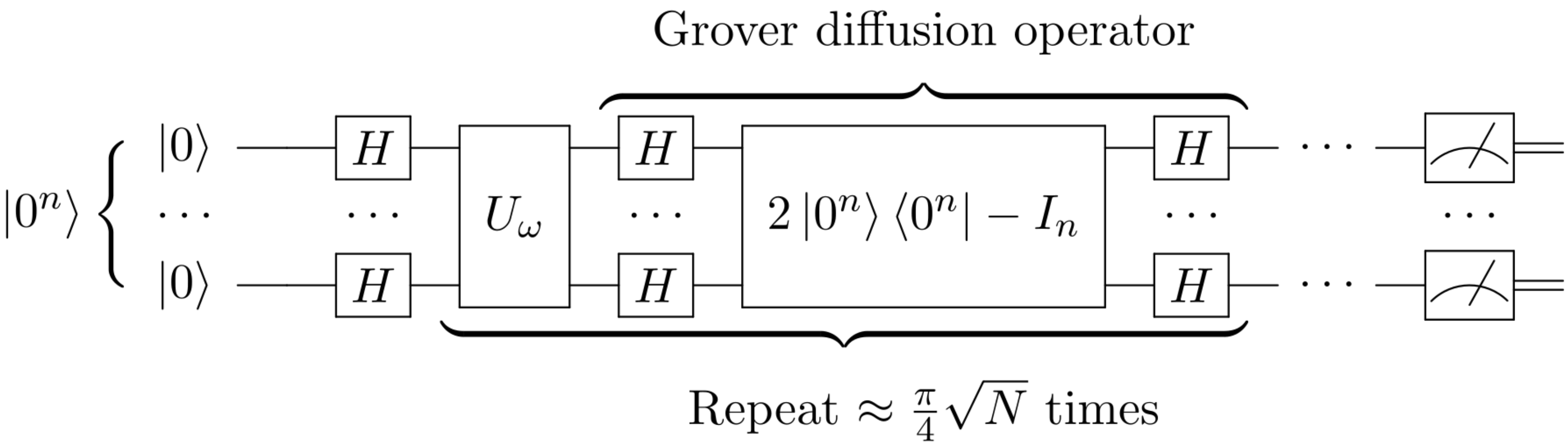


https://www.cnet.com/tech/computing/quantum-computer-makers-like-their-odds-for-big-progress-soon/

What we need for the Internet to work...

- Symmetric
  - Encryption
  - Authentication
  - Secure hashes
  - Others?
- Asymmetric
  - Encryption
  - Non-repudiability (signatures)
  - Key exchange
  - Others? (*e.g.*, homomorphic)

Grover’s algorithm



https://en.wikipedia.org/wiki/Grover%27s_algorithm#/media/File:Grover's_algorithm_circuit.svg

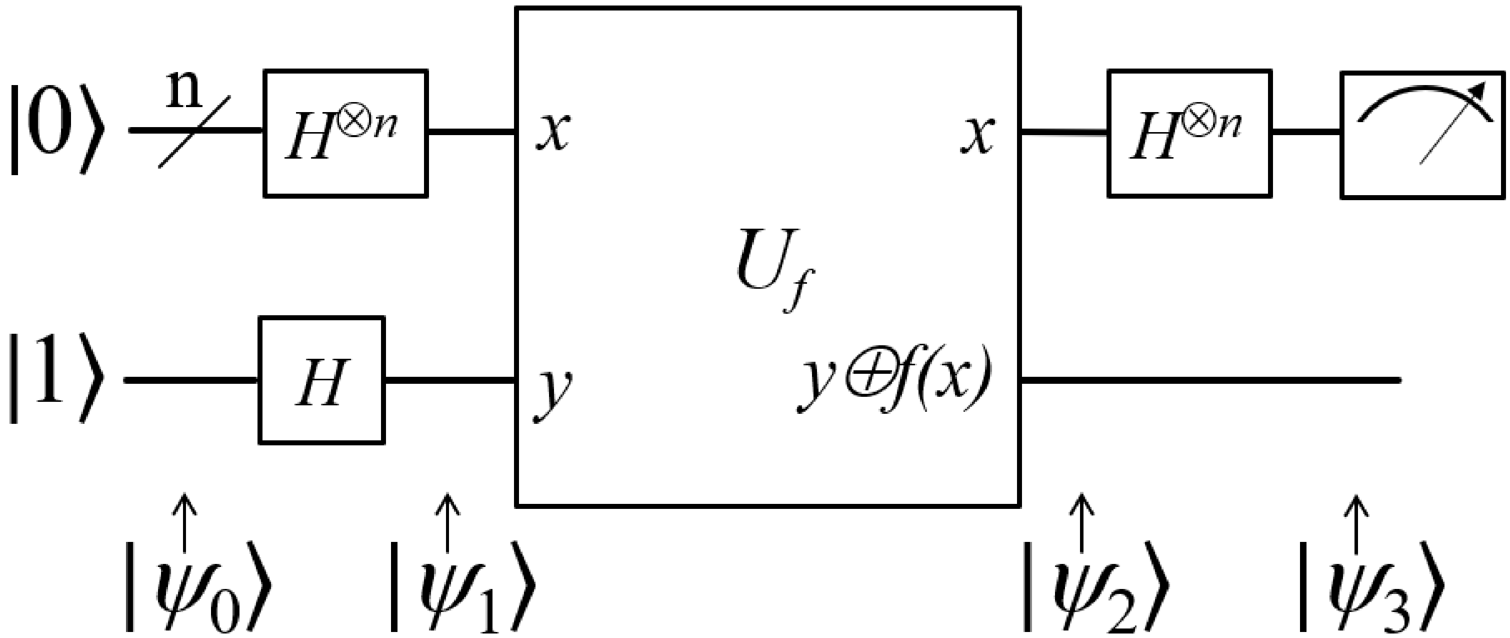
Symmetric crypto

- Just double the key size, we'll be okay (for the most part)...
  - sqrt(2^{2n}) = 2^n
  - sqrt(2^{256}) = 2^{128}

Asymmetric Crypto

- Quantum computers seem to be good at the same kinds of things that make good, simple trapdoor functions for asymmetric crypto (factorization, discrete log, *etc.*)
  - But not everything
    - Older schemes (*e.g.*, Merkle's signature scheme)
    - Newer schemes (*e.g.*, lattice-based)

Deutsh-Jozsa algorithm



https://en.wikipedia.org/wiki/Deutsch%E2%80%93Jozsa_algorithm#/media/File:Deutsch-Jozsa-algorithm-quantum-circuit.png

1-bit input case...

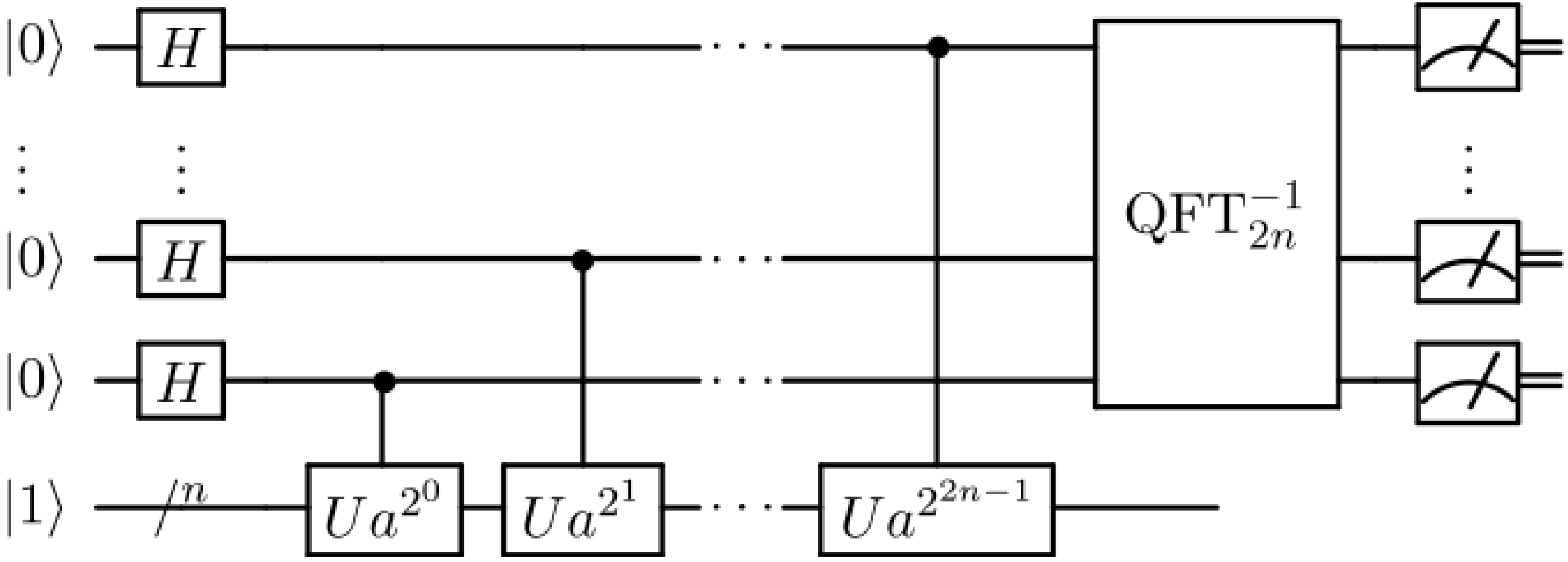
- p = Probability of measuring |0>

$$|(1/2)(-1)^{f(0)} + (1/2)(-1)^{f(1)}|$$

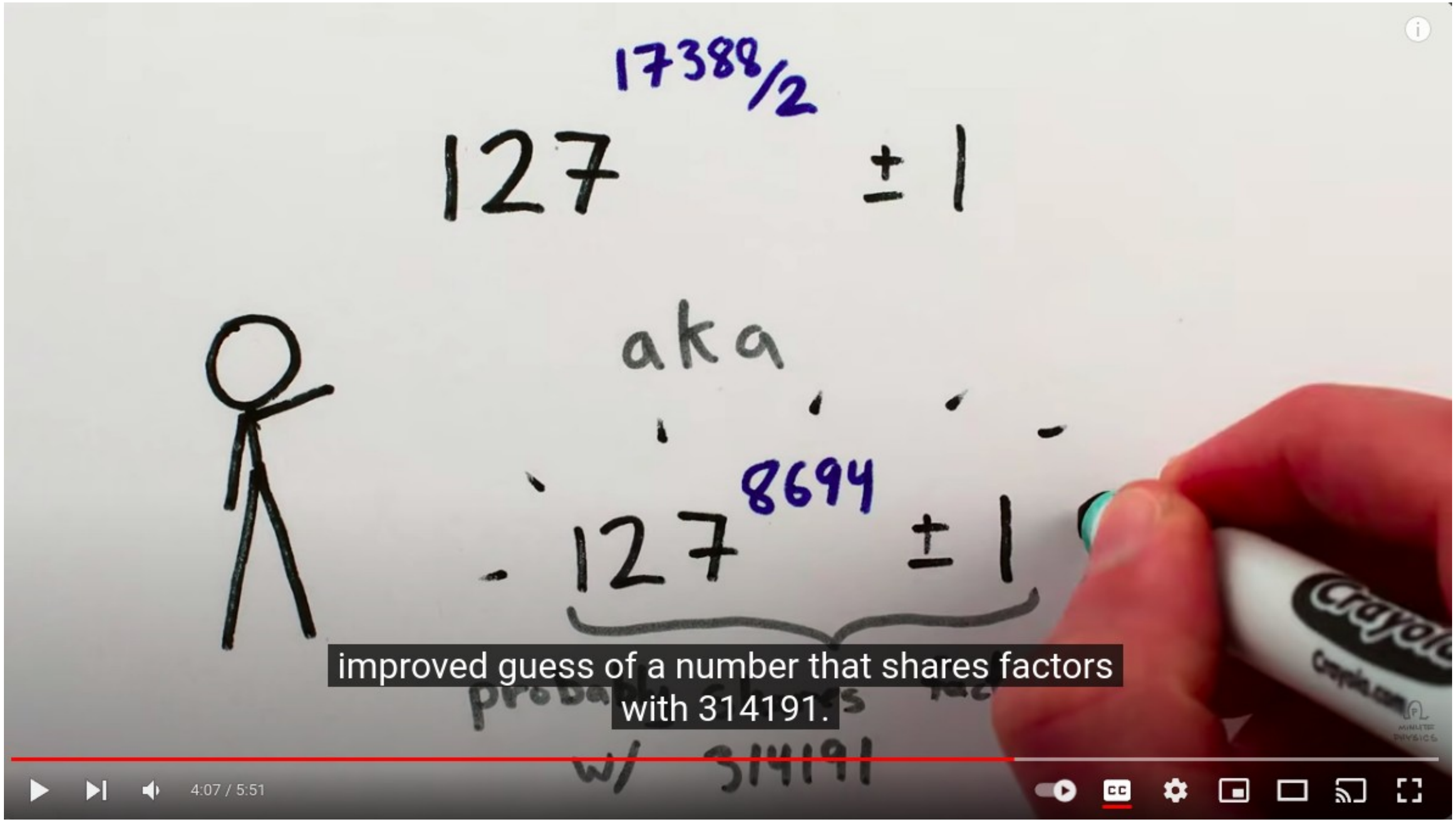
f(0)	f(1)	p
0	0	1
0	1	0
1	0	0
1	1	1

An uneven function cancels itself out because of destructive interference

Shor’s algorithm



https://en.wikipedia.org/wiki/Shor%27s_algorithm#/media/File:Shor's_algorithm.svg https://www.youtube.com/watch?v=FRZQ-efABeQ



HW 1.2 part (c)

- The sever had the private key and wouldn’t share it with the attacker, but the attacker exploited a side channel to learn the plaintext bit-by-bit
  - Whether you realized it or not, *what could have happened and didn’t* is as important to the flow of information in a padding oracle attack as what did happen
- Shor’s algorithm is a little bit like that…
  - The universe knows what the factors are
    - The wrong answers cancel each other out

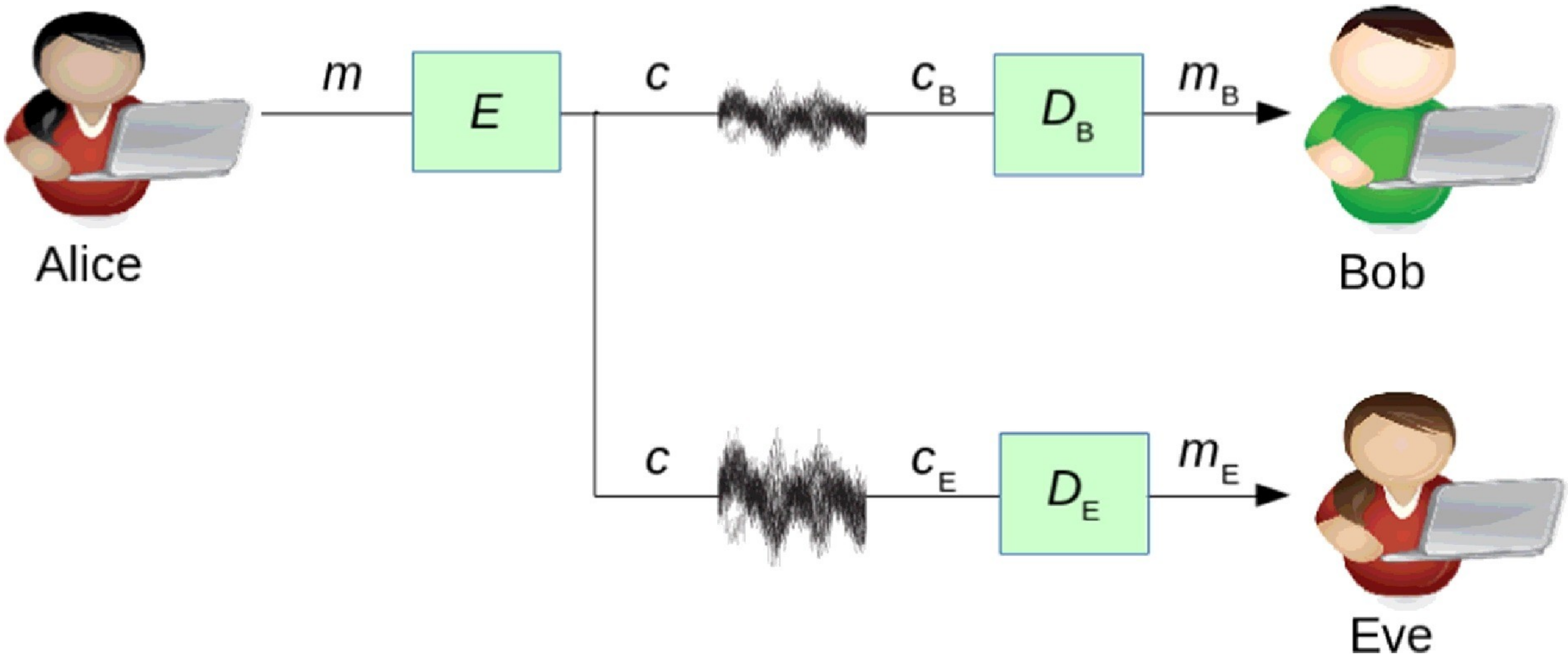
RSA, DH, ECDH, DSA, *etc*. all broken. Need something else instead...

Lamport signature (1979)

- How to sign a 256-bit message digest...
  - Generate 512 random 256-bit integers (256 pairs of them)
    - Private key
  - For all 512 generate corresponding hash
    - Public key (single use)
  - When you want to sign something, reveal one unhashed private version per pair for corresponding to the bit being 0 or 1 (*i.e.*, the first of the pair for 0, the other for 1)
    - 64 Kbits

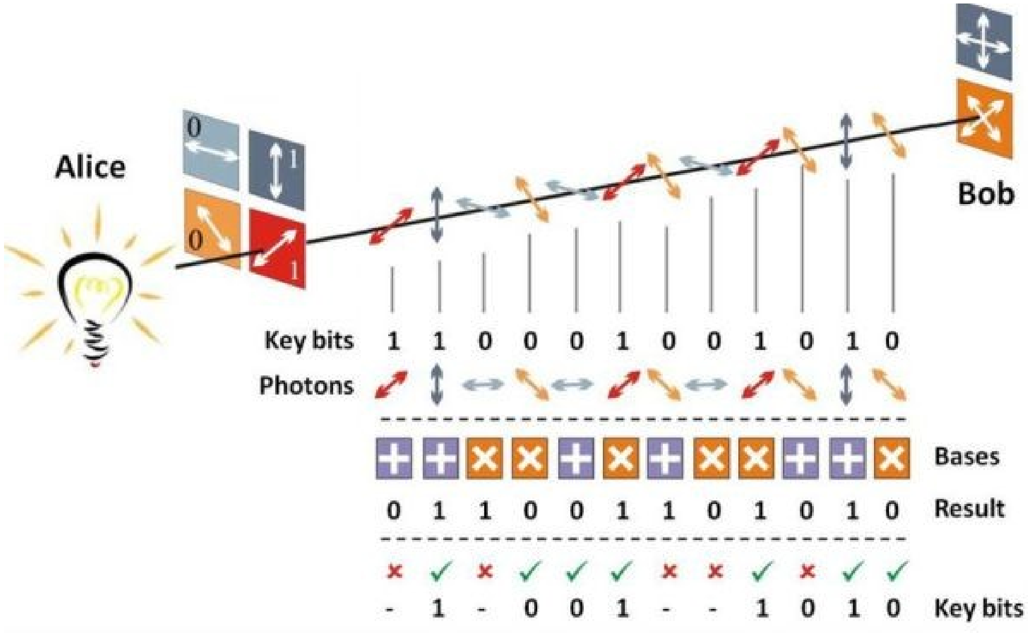
https://en.wikipedia.org/wiki/Lamport_signature

Wiretap channel



https://www.sciencedirect.com/science/article/pii/S1389128616302146

Quantum Key Distribution



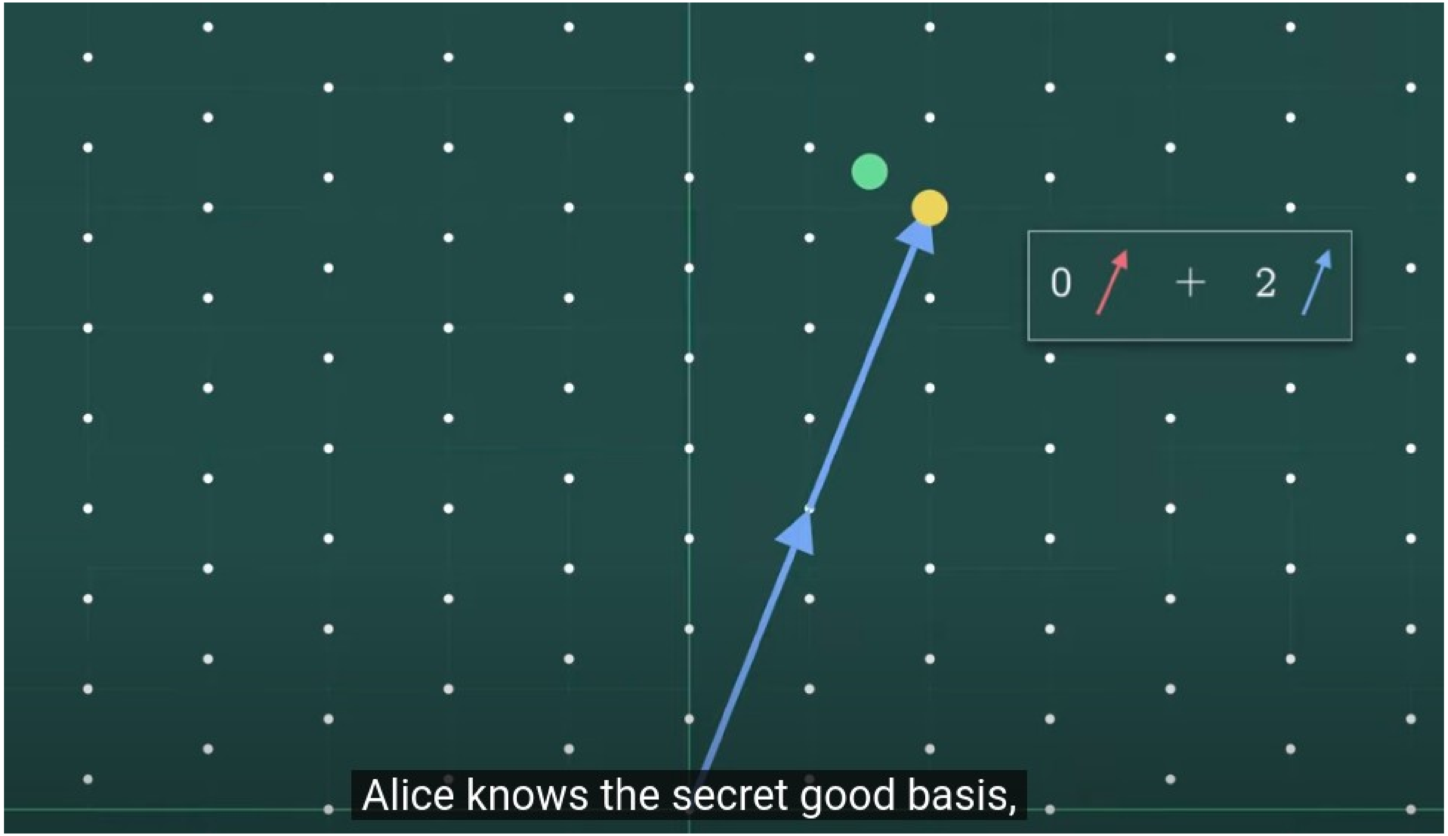
https://imrmedia.in/quantum-key-distribution-test-successfully-demonstrated/

QKD *vs.* Quantum-resistant

- QKD uses quantum physics
- Quantum-resistant crypto is performed on classical computers using one-way trapdoor functions that we *believe* will resist cryptanalysis using quantum computers

https://www.youtube.com/watch?v=QDdOoYdb748

Lattice-based cryptography: The tricky math of dots



Themes

- In schemes based on information theory or physics the eavesdropper has some noise or uncertainty the receiver doesn’t have
  - We see this in post-quantum crypto (*e.g.*, learning with errors)
- Quantum computers aren’t necessarily faster at everything
  - There’s usually a “trick at the end” where all the quantum information gets destroyed but the classical information measured still means something
  - Wrong answers cancel each other out *via* negative interference

Why do we care?

- Even schemes with perfect forward secrecy aren’t secure against a quantum computer if they’re not quantum resistant
  - Can be recorded now, broken later
- TLS, HTTPS certificates, WPA2, WPA3, 4G, 5G, WhatsApp, *etc.* are currently not “future proofed” against quantum computers
  - Signal is, but only very recently